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Published in:
Solid State Communications

DOI:
[10.1016/S0038-1098\(99\)00279-3](https://doi.org/10.1016/S0038-1098(99)00279-3)

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Document Version
Publisher's PDF, also known as Version of record

Publication date:
1999

[Link to publication in University of Groningen/UMCG research database](#)

Citation for published version (APA):

Palasantzas, G. (1999). Enhancement of proximity effects due to random roughness at a superconductor/metal interface. *Solid State Communications*, 112(2), 97-100.
[https://doi.org/10.1016/S0038-1098\(99\)00279-3](https://doi.org/10.1016/S0038-1098(99)00279-3)

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Solid State Communications 112 (1999) 97–100

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Enhancement of proximity effects due to random roughness at a superconductor/metal interface

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Received 7 April 1999; received in revised form 25 May 1999; accepted 2 June 1999 by M. Grynberg

Abstract

We consider the enhancement of proximity effects due to random roughness at a superconductor/normal-metal interface. The roughness is described by the rms roughness amplitude Δ , the correlation length ξ , and the roughness exponent H ($0 \leq H \leq 1$). Small roughness exponents H (< 0.5) are shown to influence strongly the relative reduction of the superconductor critical temperature $\Delta T_c/T_c$. Moreover, the effect of the roughness exponent H on $\Delta T_c/T_c$ appears to be more pronounced than that of the long wavelength roughness ratio σ/ξ suggesting that roughness details at short roughness wavelengths should be taken properly into account in experimental studies of proximity effects. Finally, analytic calculations of the interface roughness contribution on $\Delta T_c/T_c$ are presented for any roughness exponent $0 \leq H \leq 1$. © 1999 Elsevier Science Ltd. All rights reserved.

Keywords: A. Disordered systems; A. Heterojunctions; A. Superconductors; A. Surfaces and interfaces;

Proximity effects at the junction of a normal metal film and a thin superconducting film constitute currently a topic of intense research in the field of superconductivity (e.g. anisotropic high-temperature Cu-oxide superconductors) [1–4]. The penetration of Cooper pairs from the superconductor into the metal and electrons from the normal metal into superconductor determines the proximity effects [1,2], which manifest themselves by reducing the critical temperature T_c of a thin superconducting film covered by a thick normal metal film [1,2]. For an isotropic superconductor, if the normal superconductor/metal (SN) interface is flat the reduction of the critical temperature is given by Ref. [5] $\Delta T_c/T_c = -\gamma^2 \pi^2 \xi_0^2 / 4d^2$ with γ (≈ 0.74) a numerical factor, d the superconducting film thickness, and ξ_0 the zero temperature coherence length such that $\xi_0 \ll d$. For an anisotropic superconductor with a flat interface, the reduction of the critical temperature $|\Delta T_c|$ due to proximity effects depends on the relative orientation of the SN-interface relative to the symmetry axes of the superconductor, and becomes maximum if the SN-interface is perpendicular to

the axes corresponding to the maximum value of the coherence length. Thus, ΔT_c can be determined from the equation used for the isotropic case by replacing the squared coherence length with an average value [6].

If the SN-interface is rough, its relative orientation with respect to the symmetry axes varies influencing ΔT_c . In this case the reduction of the critical temperature for an anisotropic superconducting film is given by Ref. [6]

$$\Delta T_c/T_c = -(\gamma^2 \pi^2 \xi_c^2 / 4d^2) \{1 + [(6 + \pi^2)/3\pi^2] \times (\sum_{i=1}^2 \xi_i^2 \langle (\partial h / \partial x_i)^2 \rangle / \xi_c^2)\} \quad (1)$$

with $h(\mathbf{r})$ the SN-interface height fluctuations resulting from roughness ($h \ll d$), $\mathbf{r} = (x, y)$ is the in-plane position vector, $\partial h / \partial x_i$ are the partial height derivatives, and $\xi_{i,c}$ ($\ll d$) are the zero temperature coherence lengths along the a - b and c -axes, respectively, which are considered to be parallel to x - y and z -axes ($i = x, y, c \equiv z$). Based on Eq. (1), Mints and Snapiro [6] calculated that even moderate interface roughness ($\langle (\partial h / \partial x_i)^2 \rangle \approx 1$) can induce a considerable reduction of the critical transition temperature for coherence lengths $\xi_i \gg \xi_c$.

However, a direct quantitative relationship of interface

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roughness parameters (such as in-plane roughness correlation length ξ , rms roughness amplitude σ , and possible fractality exponents with proximity effects) is still missing. This will be the topic of the present paper where we will consider for simplicity the case of random self-affine rough interfaces. Such a rough topology is characterised by an rms roughness amplitude σ , an in-plane roughness correlation length ξ , and a roughness exponent H that describes the degree of surface irregularity at short wavelengths ($< \xi$) [7].

Measurement of the roughness parameters σ , ξ , and H has been feasible in the past by a variety of techniques such as scanning probe microscopy (atomic force and scanning tunneling microscopy), transmission electron microscopy etc. [7], X-ray reflectivity [8–11] etc. Moreover, in the experiments by Polturak et al. [4], for example, the AFM topography images would allow a direct measurement of the height–height correlation function and/or the rms roughness amplitude versus lateral scan size [7,12] yielding therefore the roughness parameters σ , ξ , and H . Other techniques to observe directly the changes in the SN-interface structure during growth of a normal metal on a superconducting film might involve e.g. X-ray reflectivity [8–11]. The latter allows a precise knowledge of the interface structure which is possibly influenced by interfacial stress/strain in contrast to the case of a bare superconducting film surface. Therefore, a more precise knowledge of the influence of roughness on the proximity effects can be gained in combination with a quantitative relation of ΔT_c on characteristic roughness parameters (e.g. σ , ξ , and H). In our study we will show quantitatively the direct implementation of roughness parameters in the formalism of proximity effects.

Initially we will rewrite Eq. (1) in a suitable form that allows direct implementation of the roughness parameters. If we define the Fourier transform of $h(\mathbf{r})$ by $h(\mathbf{q}) = \int h(\mathbf{q}) e^{-i\mathbf{q}\cdot\mathbf{r}} d^2\mathbf{q}$ and assume translation invariant interfaces or $\langle h(\mathbf{q})h(\mathbf{q}') \rangle = [(2\pi)^4/A] \langle |h(\mathbf{q})|^2 \rangle \delta^2(\mathbf{q} + \mathbf{q}')$ (with A the area of the average flat interface), Eq. (1) obtains the form

$$\Delta T_c/T_c = -(\gamma^2 \pi^2 \xi_c^2 / 4 d^2) \{1 + [(6 + \pi^2)/3\pi^2 \xi_c^2] \times (\sum_{i=1}^2 \xi_i^2 \int_{0 < q < Q_c} q_i^2 \langle |h(\mathbf{q})|^2 \rangle d^2\mathbf{q})\} \quad (2)$$

with $q^2 = \sum_{i=1,2} q_i^2$, $Q_c = \pi/a_0$ an upper roughness cut-off and a_0 of the order of the atomic spacing. Further calculation of $\Delta T_c/T_c$ from Eq. (2) will require knowledge of the roughness spectrum $\langle |h(\mathbf{q})|^2 \rangle$ which is assumed for simplicity isotropic. For isotropic self-affine fractal surfaces/interfaces, the roughness spectrum $\langle |h(\mathbf{q})|^2 \rangle$ scales as [7,13]

$$\langle |h(\mathbf{q})|^2 \rangle \propto \begin{cases} q^{-2-2H} & \text{if } q\xi \gg 1 \\ \text{const} & \text{if } q\xi \ll 1 \end{cases}, \quad (3)$$

with the roughness exponent H being a measure of the degree of surface irregularity [7,13], such that small values of H characterise more jagged or irregular surfaces at short

roughness wavelengths (ξ). Eq. (3) is satisfied by the simple Lorentzian model [14,15],¹ $\langle |h(\mathbf{q})|^2 \rangle = [A/(2\pi)^5] [\sigma^2 \xi^2 / (1 + a q^2 \xi^2)^{1+H}]$ which interpolates in a simple manner in between the asymptotic limits for large and small wave vectors, with $a = (1/2H)[1 - (1 + a Q_c^2 \xi^2)^{-H}]$ if $0 < H < 1$ and $a = (1/2)\ln[1 + a Q_c^2 \xi^2]$ if $H = 0$. This analytic roughness model will be used in the following to calculate roughness effects on $\Delta T_c/T_c$ from Eq. (2). For other roughness models [16–19] similar results are expected as in any case they will satisfy the asymptotic limits defined by Eq. (3).

Since we assume an isotropic interface roughness, Eq. (2) yields the analytic expression for the roughness contribution for any value of the roughness exponent H

$$\Delta T_c/T_c = -(\gamma^2 \pi^2 \xi_c^2 / 4 d^2) \{1 + [(6 + \pi^2)/6\pi^2 \xi_c^2] (\xi_x^2 + \xi_y^2) F(\sigma, \xi, H)\} \quad (4)$$

$$F(\sigma, \xi, H) = (\sigma^2 / 2a^2 \xi^2) \{ (1 - H)^{-1} [(1 + a Q_c^2 \xi^2)^{1-H} - 1] + H^{-1} [(1 + a Q_c^2 \xi^2)^{-H} - 1] \}.$$

For the limiting cases $H = 1$ (mound-like morphology) and $H = 0$ (logarithmic roughness) the calculation of $F(\sigma, \xi, H)$ requires the use of the identity $\lim_{s \rightarrow 0} s^{-1}(x^s - 1) = \ln x$. Thus we obtain $F = (\sigma^2 / 2a^2 \xi^2) \{ a Q_c^2 \xi^2 - \ln(1 + a Q_c^2 \xi^2) \}$, and $F = (\sigma^2 / 2a^2 \xi^2) \{ \ln(1 + a Q_c^2 \xi^2) + a Q_c^2 \xi^2 \}$, respectively for $H = 0$ and $H = 1$.

Our calculations of $\Delta T_c/T_c$ were performed via Eq. (4) for film thickness $d = 50$ nm, coherence lengths $\xi_x = \xi_y = 2$ nm, and $\xi_c = 0.3$ nm [4,6] such that $\xi_{x,y,c} \ll d$ and $\xi_c \ll \xi_{x,y}$ [4,6], $a_0 = 0.3$ nm, and rms roughness amplitude $\sigma = 5$ nm ($\sigma \ll d$). The relative reduction in critical temperature $\Delta T_c/T_c$ has a trivial dependence on the rms roughness amplitude σ ; namely $\Delta T_c/T_c \propto \sigma^2$, while any complex dependence will arise solely from the roughness exponent H and the in-plane correlation length ξ . Fig. 1 shows the absolute reduction in temperature $|\Delta T_c|/T_c$ versus the roughness exponent H for a variation of the correlation length ξ by an order of magnitude (such that $\sigma/\xi < 0.1$). As the roughness exponent H increases from 0 to 1, $|\Delta T_c|/T_c$ can decrease by more than two orders in magnitude. Nevertheless, the highest reduction in critical temperature (up to $\sim 20\%$) occurs for small roughness exponents ($H < 0.5$) indicating that the more irregular is the interface at short wavelengths the larger is the roughness influence on the proximity effects and thus on the critical temperature T_c . An increase of the correlation length ξ by an order of magnitude (or equivalently a decrease of the ratio σ/ξ for fixed roughness amplitude σ ; smoothing at long wavelengths) leads obviously to a smaller decrease of $|\Delta T_c|/T_c$.

Fig. 2 shows the direct dependence of $|\Delta T_c|/T_c$ on the

¹ Besides the simplicity of $\langle |h(\mathbf{q})|^2 \rangle$, its Fourier transform yields the analytically solvable correlation function $C(r) = [\sigma^2 / (ocr(1+H))]^2 a \Gamma(1+H) / (r/2a^{1/2} \xi)^H K_H(r/2a^{1/2} \xi)$ with $K_\nu(x)$ the second kind Bessel function of order ν .

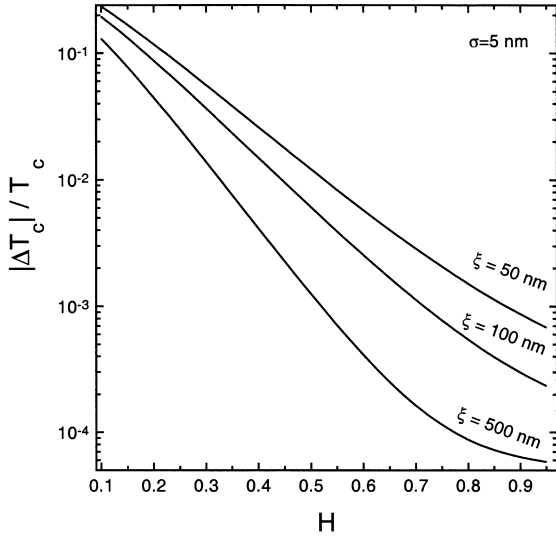


Fig. 1. Relative absolute reduction of critical temperature $|\Delta T_c|/T_c$ versus the roughness exponent H and the other parameters as indicated with $\sigma/\xi < 0.1$.

correlation length ξ for fixed roughness amplitude σ . Although the curves indicate $|\Delta T_c|/T_c$ to decrease relatively more for large roughness exponents ($H > 0.5$; smoother surfaces at short wavelengths), as in absolute magnitude the relevant decrement of $|\Delta T_c|/T_c$ (~ 1 –20%) occurs at small roughness exponents ($H < 0.5$). Therefore, in comparison with Fig. 1, it is clearly evident that the

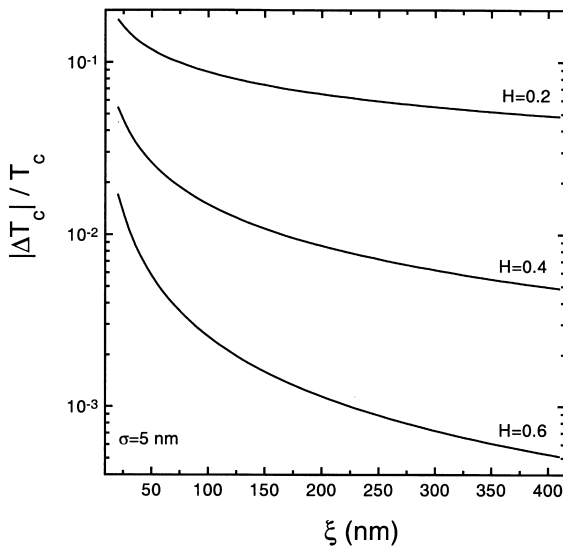


Fig. 2. Relative absolute reduction of critical temperature $|\Delta T_c|/T_c$ versus the roughness correlation length ξ and the other parameters as indicated. The effect of the correlation length ξ (or equivalently of the ratio σ/ξ for σ fixed) is less pronounced than that of H .

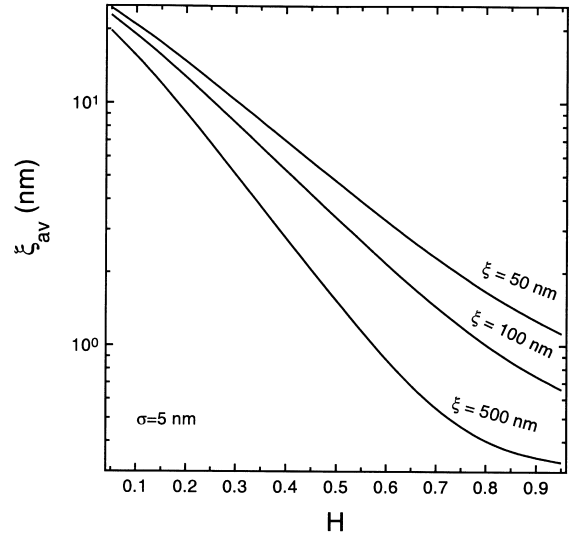


Fig. 3. Average coherence length ξ_{av} versus the roughness exponent H and the other parameters as indicated with $\sigma/\xi < 0.1$. ξ_{av} can decrease by two orders of magnitude as H increases from 0 to 1.

strongest influence on the proximity effects comes from the roughness exponent H or alternatively from fine roughness details at short wavelengths ($< \xi$) which can be enhanced significantly for large long wavelength roughness ratios σ/ξ (~ 0.1).

Eq. (4), in comparison with equation $\Delta T_c/T_c = -\gamma^2 \pi^2 \xi_0^2 / 4d^2$ used for an isotropic superconducting film (with a flat SN-interface), yields an average zero-temperature coherence length $\xi_{av} (> \xi_c)$

$$\xi_{av} = \{ \xi_c^2 + [(6 + \pi^2)/6\pi^2](\xi_x^2 + \xi_y^2)F(\sigma, \xi, H) \}^{1/2} \quad (5)$$

that incorporates interface roughness effects. Fig. 3 depicts the dependence of the average coherence length ξ_{av} on the roughness exponent H for various correlation lengths ξ (such that $\sigma/\xi < 0.1$). It is clearly evident that the dominant contribution arises from the exponent H (which is associated with a local fractal dimension $D = 3 - H$ [7,8–11]). For small roughness exponent $H (< 0.5)$, ξ_{av} can be larger than ξ_c closely by two orders of magnitude, while for large roughness exponents ($H \sim 1$) and small roughness ratios $\sigma/\xi (< 0.1)$ the average coherence length ξ_{av} approaches asymptotically ξ_c (Fig. 3).

In conclusion, we investigated the influence of interface roughness on proximity effects induced at the junction of a thick normal metal and a superconducting thin film. It is shown that the proximity effect to be influenced predominantly (with respect to long wavelength roughness ratio σ/ξ) by the degree of interface irregularity at short wavelengths as expressed by the roughness exponent H . Similar conclusions were drawn also for the average coherence length ξ_{av} that incorporates interface roughness effects. Therefore, in future studies of rough SN-junctions the precise roughness nature at all roughness wavelengths

should be properly quantified (measurement of H , σ , and ξ , e.g., by X-ray reflectivity, scanning probe microscopy etc.) in order to gauge precisely morphology contributions on proximity effects.

Acknowledgements

I would like to acknowledge support from the Department of Applied Physics at Delft University of Technology and useful discussions with L.J. Geerligs.

References

- [1] P.G. de Gennes, *Superconductivity of Metals and Alloys*, Benjamin, New York, 1966.
- [2] G. Deutscher, P.G. de Gennes, in: R.D. Parks (Ed.), *Superconductivity*, 2, Marcel Dekker, New York, 1969.
- [3] K.A. Delin, A.W. Kleinsasser, *Supercond. Sci. Technol.* 9 (1996) 227.
- [4] E. Polturak, G. Koren, D. Coden, O. Neshet, R.G. Mints, I. Snapiro, *Phys. Rev. B* 57 (1998) 14068.
- [5] A.A. Abrikosov, *Fundamentals of the Theory of Metals*, North-Holland, Amsterdam, 1988.
- [6] R.G. Mints, I.B. Snapiro, *Phys. Rev. B* 57 (1998) 10318.
- [7] J. Krim, G. Palasantzas, *Int. J. Mod. Phys. B* 9 (1995) 599.
- [8] R. Paniago, R. Forest, P.C. Chow, S.C. Moss, S.P. Parkin, D. Cookson, *Phys. Rev. B* 56 (1997) 13442.
- [9] S.K. Sinha, E.B. Sirota, S. Garoff, H.B. Stanley, *Phys. Rev. B* 38 (1988) 2297.
- [10] V. Holy, J. Kubena, I. Ohlidal, K. Lischka, W. Plotz, *Phys. Rev. B* 47 (1993) 15896.
- [11] M. Chladek, V. Volvoda, C. Dorner, C. Holy, J. Grim, *Appl. Phys. Lett.* 69 (1996) 1318.
- [12] J. Aue, J.Th.M. De Hosson, *Appl. Phys. Lett.* 71 (1997) 1347 For AFM tip effects on the measurement of the roughness exponent H see for example (and references therein).
- [13] P. Meakin, *Phys. Rep.* 235 (1993) 1991.
- [14] G. Palasantzas, *Phys. Rev. B* 48 (1993) 14472.
- [15] G. Palasantzas, *Phys. Rev. B* 49 (1994) 5785 E.
- [16] H.-N. Yang, T.-M. Lu, *Phys. Rev. E* 51 (1995) 2479.
- [17] Y.-P. Zhao, G.C. Wang, T.-M. Lu, *Phys. Rev. B* 55 (1997) 13938.
- [18] G. Palasantzas, J. Krim, *Phys. Rev. B* 48 (1993) 2873.
- [19] G. Palasantzas, *Phys. Rev. E* 49 (1994) 1740.